Reg.No.:								

VIVEKANANDHA COLLEGE OF ENGINEERING FOR WOMEN
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Question Paper Code: 2002

B.E. / B.Tech. DEGREE SUPPLEMENTARY EXAMINATIONS – FEB. / MAR. 2020
Third Semester
Electrical and Electronics Engineering
U15MA304 – COMPLEX ANALYSIS AND PARTIAL DIFFERENTIAL EQUATIONS
(Common to Electronics and Communication Engineering & Biotechnology)
(Regulation 2015)

Time : Three Hours Maximum : 100 Marks

Answer ALL the questions

PART – A

(10 x 2 = 20 Marks)

1. Verify whether $f(z)=\bar{z}$ is analytic or not?
2. Find the invariant points of the transformation $w=\frac{z-1}{z+1}$.
3. Evaluate: $\int_C \frac{z}{z-2}dz$, where C is the circle $|z|=1$.
4. State Cauchy's Residue theorem.
5. State Dirichlet's conditions for Fourier series.
6. Determine the value of a_n in the Fourier series expansion of $f(x)=x^3$ in $-\pi<x<\pi$.
7. Find the Fourier cosine transform of $f(x)=e^{-ax}$ $(a>0)$.
8. Define Fourier transform pair.
9. Form a p.d.e by eliminating the arbitrary constants a and b from $z=ax^2+by^2$
10. Classify the p.d.e $3u_{xx}+10u_{xy}+3u_{yy}=0$.

PART – B

(5 x 16 = 80 Marks)

11. a) i. If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ (8)

ii. Show that $\frac{1}{2}\log(x^2+y^2)$ is harmonic. Determine the analytic function. Find also its Conjugate (8)

(OR)

b) i. Find the bilinear transformation which maps the points $z = 0, 1, \infty$ into $w = i, 1, -i$ respectively. (8)

ii. Find the image of $|z-2i| = 2$ under the transformation $w = \frac{1}{z}$. (8)

12. a) i. Evaluate: $\int_C \frac{z}{(z-1)(z-2)^2}dz$ where c is the circle $|z-2| = \frac{1}{2}$, by using Cauchy's integral formula. (8)

ii. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ as a Laurent's series valid in the region $1 < |z| < 3$ (8)

(OR)

b) i. Using contour integration, evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ (8)

ii. Evaluate $\int_c \left(\frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\right)dz$, *where c is* $|z| = 3$. (8)

13. a) Find the Fourier series for the function $f(x) = x(2\pi - x)$ *in* $(0,2\pi)$ and hence deduce that sum of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots = \frac{\pi^2}{6}$.

(OR)

b) i. Find the half range sine series of $f(x) = lx - x^2 in(0, l)$ (8)

ii. Find the Fourier series upto second harmonic for the following data: (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

14. a) Find the Fourier transform of $f(x)=\begin{cases}1-|x| & if\ |x|<1\\ 0 & if\ |x|>1\end{cases}$

and hence evaluate

i. $\int_0^\infty \left(\frac{\sin^4 t}{t^4}\right)\ dt$

ii. $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$

(OR)

b) i. Prove that $e^{-x^2/2}$ is a self-reciprocal under Fourier transform (8)

ii. Use transform method to evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$. (8)

15. a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x,0) = K(lx-x^2)$. It is released from rest from this position. Find the expression for the displacement at any time 't'.

(OR)

b) A rod 30 cm long has its ends A and B kept at 20^0 and 80^0 respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^0 and kept so. Find the resulting temperature function $u\ (x, t)$ taking $x = 0$ at A.